Motivating Question
Find a diagrammatic basis for the space of invariant tensors \( \text{Irr}(V^{\otimes n}) \) that respects the natural cyclic action of \( C_n \) and where \( V \) is:
- the spin representation of type \( B \)
- the vector representation of type \( C \)
- the vector representation of type \( B \) (not shown on poster)

Highest Weight Words of Weight Zero
Cyclic action on invariant tensors of \( V^{\otimes n} \) → promotion on \( \text{HW}_{n2} \) in crystal \( B_{n/2} \) [12].
- \( r \)-fans of Dyck paths of length \( n \) → \( \text{HW}_{n2} \) in \( \text{Spin} \) of type \( B_n \)
- \( r \)-symplectic oscillating tableaux of length \( n \) and weight \( \mu \) → \( \text{HW}_{n2} \) in \( \text{Spin} \) of type \( C_n \)
- \( \lambda \)-triple \((\gamma, \kappa, \nu, \mu) \)

Chord Diagrams via Promotion

Promotion (via local rules) and Filling Rule

The local rules of Lenart [7] can be stated as follows: four weight vectors \( \lambda, \mu, \kappa, \nu \in \Lambda \) depicted in a square diagram satisfy the local rule, if \( \mu = \text{dom}(\kappa + \nu - \lambda) \). Promotion on highest weight elements in minuscule crystals can be defined in the same local rules as shown below.

The filling rule for oscillating tableaux and \( r \)-fans of Dyck path is

\[ F(\lambda, \kappa, \nu, \mu) = \text{number of negative entries in } \kappa + \nu - \lambda \]

Chord Diagrams from Fomin Growth Diagrams
Given a square diagram with filling, the local growth rules for oscillating tableaux are given by:

- (F1) \( \beta^i = \text{sort}(\alpha^i + \gamma^i - \chi^i) \) if the filling is 0
- (F2) \( \beta = \gamma + \epsilon_j \) if the filling is 1.

To construct the adjacency matrix \( G(C) \) or \( G(F) \):
1. Label the hypotensis of a staircase Ferrer shape with \( O \) or \( F \) respectively.
2. Apply the inverse local growth rules.
3. Turn the growth diagram into a symmetric matrix.

Promotion and Growth Diagrams for Fans of Dyck Paths

Promotion (via local rules) and Filling Rule
- \( r \)-fans of Dyck paths of length \( n \) → \( \text{HW}_{n2} \) in \( \text{Spin} \) of type \( B_n \)
- \( r \)-symplectic oscillating tableaux of length \( n \) and weight \( \mu \) → \( \text{HW}_{n2} \) in \( \text{Spin} \) of type \( C_n \)
- \( \lambda \)-triple \((\gamma, \kappa, \nu, \mu) \)

Chord Diagrams via Promotion

Promotion (via local rules) and Filling Rule

The local rules of Lenart [7] can be stated as follows: four weight vectors \( \lambda, \mu, \kappa, \nu \in \Lambda \) depicted in a square diagram satisfy the local rule, if \( \mu = \text{dom}(\kappa + \nu - \lambda) \). Promotion on highest weight elements in minuscule crystals can be defined in the same local rules as shown below.

The filling rule for oscillating tableaux and \( r \)-fans of Dyck path is

\[ F(\lambda, \kappa, \nu, \mu) = \text{number of negative entries in } \kappa + \nu - \lambda \]

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References