The above conjecture is true for $r = 1$ and $r = 2$ with $X_0 = 0$ and $X_1 = 7$, respectively.

Theorem 1. The above conjecture is true for $r = 2$ and $t = 2$ with $X_0 = 0$ and $X_1 = 7$, respectively.

Proof outline and remarks.

- We first find the generating functions $a_2(n)$, $b_2(n)$, $a_3(n)$, $b_3(n)$.
- The generating function for $a_2(n) - b_2(n)$ leads to a new combinatorial interpretation of $a_2(n) - b_2(n)$ (see the eight columns which will be evidenced non-negative, showing $-b_2(n)$ for all $n = 0, 1$).

The main result is established in Theorem 1. In [3], we prove the conjecture for $t = 2$ and $r = 1$.

The total number of partitions of $n$ is the generating function for the number of distinct partitions of $n$ with a hook of type $I$. The total number of distinct partitions of $n$ is $D_n$. Let $D_n$ denote the number of distinct partitions of $n$. Let $a_2(n)$ denote the number of hook of length $1$ in a partition $\lambda$. Let $a_2(n) = \left| a_2(n) \right|$, $b_2(n) = \left| b_2(n) \right|$, and $a_3(n) = \left| a_3(n) \right|$. For a positive integer, let $\sigma(n)$ denote the number of part $\lambda$.

Let $h_0, h_1, h_2$ denote the number of hooks of length $0, 1, 2, 3$ in a partition $\lambda$, respectively.

The total number of partitions of $n$ is the generating function for the number of distinct partitions of $n$ with a hook of type $I$. The total number of distinct partitions of $n$ is $D_n$.

Theorem 2. Suppose $\sum a_2(n)q^n \subset \sum b_2(n)q^n \subset \sum a_3(n)q^n$.

The generating function for the number of distinct partitions of $n$ with a hook of type $I$. The total number of distinct partitions of $n$ is $D_n$.

The total number of partitions of $n$ is the generating function for the number of distinct partitions of $n$ with a hook of type $I$. The total number of distinct partitions of $n$ is $D_n$.

Theorem 2. Suppose $\sum a_2(n)q^n \subset \sum b_2(n)q^n \subset \sum a_3(n)q^n$.

The generating function for the number of distinct partitions of $n$ with a hook of type $I$. The total number of distinct partitions of $n$ is $D_n$.

The total number of partitions of $n$ is the generating function for the number of distinct partitions of $n$ with a hook of type $I$. The total number of distinct partitions of $n$ is $D_n$.

Theorem 2. Suppose $\sum a_2(n)q^n \subset \sum b_2(n)q^n \subset \sum a_3(n)q^n$.

The generating function for the number of distinct partitions of $n$ with a hook of type $I$. The total number of distinct partitions of $n$ is $D_n$.

The total number of partitions of $n$ is the generating function for the number of distinct partitions of $n$ with a hook of type $I$. The total number of distinct partitions of $n$ is $D_n$.

Theorem 2. Suppose $\sum a_2(n)q^n \subset \sum b_2(n)q^n \subset \sum a_3(n)q^n$.

The generating function for the number of distinct partitions of $n$ with a hook of type $I$. The total number of distinct partitions of $n$ is $D_n$.

The total number of partitions of $n$ is the generating function for the number of distinct partitions of $n$ with a hook of type $I$. The total number of distinct partitions of $n$ is $D_n$.

Theorem 2. Suppose $\sum a_2(n)q^n \subset \sum b_2(n)q^n \subset \sum a_3(n)q^n$.

The generating function for the number of distinct partitions of $n$ with a hook of type $I$. The total number of distinct partitions of $n$ is $D_n$.

The total number of partitions of $n$ is the generating function for the number of distinct partitions of $n$ with a hook of type $I$. The total number of distinct partitions of $n$ is $D_n$.